Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
Mechanics (9FM0) Paper 4C

Edexcel and BTEC Qualifications

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Summer 2019
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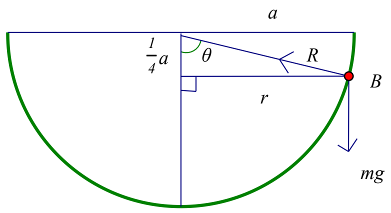
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Answer	Mark
1	**1. The only correct answer is A** ***B*** *is not correct because cash in hand is an asset* ***C*** *is not correct because debtors are an asset* ***D*** *is not correct because prepaid expenses are an asset*	**(4)**

Question	Scheme	Marks	AOs
1			
	$\updownarrow R\cos\theta = mg$	M1	3.1b
	$\leftrightarrow R\sin\theta = mr\omega^2$	M1	3.3
		A1	1.1b
	$\tan\theta = \dfrac{r}{a/4}$ $\quad\left(\tan\theta = \sqrt{15}\right)$	B1	1.1b
	Complete strategy to find ω	M1	3.1b
	$\tan\theta = \dfrac{mr\omega^2}{mg} = \dfrac{4r}{a}$, $\Rightarrow \omega^2 = \dfrac{4g}{a}$, $\omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
		(6)	
	Alternative:		
	$\updownarrow R\cos\theta = mg$	M1	3.1b
	$\leftrightarrow R\sin\theta = ma\sin\theta\omega^2$	M1	3.3
		A1	1.1b
	$\cos\theta = \frac{1}{4}$	B1	1.1b
	Complete strategy to find ω	M1	3.1b
	$\Rightarrow R = 4mg,\ 4mg = ma\omega^2 \Rightarrow \omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
		(6 marks)	

Question	Marks	Marking Guidance
1		Check their diagram to see where they have put θ Check the working carefully, particularly the value of r: some errors in the working can lead to a fortuitously correct answer
	M1	Resolve vertically. Must be dimensionally correct.

	M1	Resolve horizontally and form equation for circular motion. Must be dimensionally correct.
	A1	Correct pair of equations for their unknowns (any *r*)
	B1	Correct trig ratio(s) seen or implied. Allow for $r = \frac{\sqrt{15}}{4}a$
	M1	Complete strategy to form and solve a set of equations with $r \neq a$ to find ω. For solving their 2 equations – not dependent
	A1	Eliminate additional variables to obtain ω. Accept equivalent exact forms.
	(6)	
		If *R* does not act through the centre of the hemisphere then the maximum available is M1M1A0B0M0A0: 2/6

Question	Scheme	Marks	AOs
2(a)	Strategy to find v^3 in terms of x	M1	3.1a
	Differential equation in v and x: $0.4v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{k}{v}$	M1	2.1
	$\Rightarrow 0.4v^2\frac{\mathrm{d}v}{\mathrm{d}x} = k\ ,\quad \frac{0.4}{3}v^3 = kx + C$	A1	1.1b
	$x = 3,\ v = 2\quad \frac{3.2}{3} = 3k + C$ $x = 6,\ v = 2.5\quad \frac{25}{12} = 6k + C$	M1	2.1
	$\Rightarrow 3k = \frac{25}{12} - \frac{3.2}{3}\ ,\quad k = \frac{61}{180}\ ,\quad C = \frac{1}{20}$	A1	1.1b
	$v^3 = \frac{3}{0.4}\left(\frac{61x}{180} + \frac{1}{20}\right) = \frac{61x+9}{24}\quad *$	A1*	2.2a
		(6)	
(b)	$\frac{5k}{2v} = \frac{\mathrm{d}v}{\mathrm{d}t} = \left(\frac{61}{72v}\right)$	M1	2.5
	$\int 2v\mathrm{d}v = \int 5k\mathrm{d}t\ \Rightarrow\ v^2 = 5kt + C'\quad \left(36v^2 = 61t + C'\right)$	M1	2.1
	$\left[v^2\right]_2^{2.5} = \left[5kt\right]_0^T$ $\left(61T = 36\left(2.5^2 - 2^2\right)\right)$	M1	1.1b
	$T = \frac{180}{61}\left(\frac{9}{20}\right) = \frac{81}{61}\quad *$	A1*	2.2a
		(4)	
(b) alt	$\frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt[3]{\frac{61x+9}{24}}$	M1	2.5
	$\int (61x+9)^{-\frac{1}{3}}\mathrm{d}x = \int \frac{1}{\sqrt[3]{24}}\mathrm{d}t\ ,\quad \frac{3}{2\times 61}(61x+9)^{\frac{2}{3}} = \frac{t}{\sqrt[3]{24}} + C''$	M1	2.1
	$T = 2\times\sqrt[3]{3}\times\frac{3}{2\times 61}\left(375^{\frac{2}{3}} - 192^{\frac{2}{3}}\right) = \frac{3\times\sqrt[3]{3}}{61}\left(\left(5\sqrt[3]{3}\right)^2 - \left(4\sqrt[3]{3}\right)^2\right)$	M1	1.1b
	$T = \frac{9}{61}(25-16) = \frac{81}{61}\quad *$	A1*	2.2a
(10 marks)			

Question	Marks	Marking Guidance
2(a)	M1	Complete strategy e.g. use of $F = ma$ with appropriate form for a, and use boundary conditions to confirm given result
	M1	Separate variables and integrate. Usual rules for integration. Condone missing C
	A1	Correct integration. Accept equivalent forms. Condone missing C.
	M1	Use boundary conditions to form 2 equations in 2 unknowns and solve for k or C
	A1	Obtain correct values for the constants
	A1*	Obtain **given answer** from correct working
	(6)	
(b)	M1	Select correct form for derivative and form a correct differential equation in v and t – follow their k
	M1	Separate and integrate. Condone with no $+C'$ – follow their k
	M1	Evaluate definite integral of the form $pv^2 = qt + C'$ or use limits to find value of constant of integration – follow their k
	A1*	Obtain **given answer** from correct working
	(4)	
(b) alt	M1	Select correct form for derivative and form a correct differential equation in x and t
	M1	Separate and integrate. Condone with no $+C''$
	M1	Evaluate definite integral of the form $pt = q\left(61x+9\right)^{\frac{2}{3}} + C''$ or use limits to find value of constant of integration
	A1*	Obtain **given answer** from correct working
		NB: Both parts have given answers, so check very carefully

Question	Scheme	Marks	AOs
3(a)	Correct strategy	M1	3.1a
	$8\bar{y}=\frac{1}{2}\int y^2\mathrm{d}x=\frac{1}{2}\int\left\{\frac{(x-2)^6}{16}+(x-2)^3+4\right\}\mathrm{d}x$	M1	2.1
	$=\frac{1}{2}\left[\frac{(x-2)^7}{7\times16}+\frac{(x-2)^4}{4}+4x\right]_0^4$	A1	1.1b
	$8\bar{y}=\frac{1}{2}\left[\frac{8}{7}+4+16+\frac{8}{7}-4-0\right]=\frac{64}{7}$, $\bar{y}=\frac{8}{7}$ *	A1*	2.2a
		(4)	
(b)	$8\bar{x}=\int\left(\frac{x(x-2)^3}{4}+2x\right)\mathrm{d}x$	M1	2.1
	$=\left[\frac{x(x-2)^4}{16}-\frac{(x-2)^5}{80}+x^2\right]_0^4$	A1	1.1b
	$=\frac{64}{16}-\frac{32}{80}+16-\frac{32}{80}=19.2$	M1	1.1b
	$\bar{x}=2.4$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan\theta=\frac{4-\bar{x}}{4-\frac{8}{7}}\left(=\frac{14}{25}\right)$	A1ft	3.4
	$\theta=29.2$ (Accept 29)	A1	1.1b
		(7)	
			(11 marks)

Question	Marks	Marking Guidance
3(a)	M1	Complete strategy for $\bar{y}$: moments equation, use of limits and division by area
	M1	Moments equation to obtain terms of the correct form (with or without limits) The integral must be in terms of x only or y only Allow if area (8) not seen Might see $\int\frac{x^6}{16}-\frac{3x^5}{4}+\frac{15x^4}{4}-9x^3+9x^2\mathrm{d}x$

		Or $\int xy\mathrm{d}y = \int 2y + y\left(4\left(y-2\right)\right)^{\frac{1}{3}}\mathrm{d}y$
	A1	Correct unsimplified answer (with or without limits) Allow if area (8) not seen $\left(\int xy\mathrm{d}y = 12.8\right)$
	A1*	Use moments equation and given area to deduce **given answer** from correct working
	(4)	
(b)	M1	Relevant integral in terms of x only or y only (with or without limits). Allow if area (8) not seen Could start with $\int xy\mathrm{d}x$ or $\int \frac{1}{2}x^2\mathrm{d}y$ Might see $\frac{x^4}{4} - \frac{6x^3}{4} + 3x^2 - 2x + 2x$
	A1	Correct unsimplified form after integration (with or without limits). Allow if area (8) not seen
	M1	Complete process to find $\bar{x}$ following relevant integral
	A1	Correct answer
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 29.2488...
	(7)	

Question	Scheme	Marks	AOs
4(a)	Total mass $=\int_0^4 (18-3x)\,\mathrm{d}x$	M1	2.1
	$=\left[18x-\frac{3x^2}{2}\right]_0^4$	A1	1.1b
	$=18\times4-\frac{3\times16}{2}\ \ (=72-24)\ \ =48\ \ (\mathrm{kg})\ \ *$	A1*	1.1b
		(3)	
(b)	Taking moments about the base: $\int_0^4 x(18-3x)\,\mathrm{d}x$	M1	3.4
	$=\left[9x^2-x^3\right]_0^4(=80)$	A1	1.1b
	$\Rightarrow 48d=80$	M1	3.4
	$d=\frac{80}{48}=\frac{5}{3}$ (m)	A1	1.1b
	Complete strategy	M1	3.1b
	$\mathrm{M}(A): 2T=4\cos45^\circ\times4g+\frac{5}{3}\cos45^\circ\times48g$ $\left(=\frac{96g}{\sqrt{2}}\right)$	A1ft A1ft	1.1b 1.1b
	$T=333$ or 330	A1	2.2a
		(8)	
(c)	Any appropriate comment e.g. the ball has been modelled as a point mass – its centre could be further from *A*	B1	3.5b
		(1)	
			(12 marks)

Question	Marks	Marking Guidance
4(a)	M1	Use integration (usual rules) – do not need to see limits at this stage
	A1	(M1 on epen) Correct integration and correct limits seen
	A1*	Show sufficient working to justify given answer
	(3)	

(b)	M1	Use the model to find the moment of the pole and the ball about *A* (usual rules for integration)
	A1	Correct integration
	M1	Use the model to complete the moments equation. Require their 80 and 48 used correctly
	A1	Any equivalent form
	M1	Complete strategy to find the tension – e.g.locate c of m of the pole and use moments.
	A1ft A1ft	Moments equation with at most one error. Follow their c of m provided not at centre Correct unsimplified moments equation for their c of m not at centre
	A1	Accept $24\sqrt{2}g$, 333 or 330 ISW
	(8)	
(c)	B1	• The mass of the cable has been ignored – unlikely to be negligible if it can hold a pole this long. • The flagpole will be subject to cross winds • The cable might be extensible • The pole might not be rigid NOT: the wall might be rough / smooth Ignore incorrect statements
	(1)	

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>$\int \pi x^2 y\mathrm{d}y = \pi\int \frac{y^5}{4}\mathrm{d}y = \pi\left[\frac{y^6}{24}\right]_0^2$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>$= \frac{64\pi}{24}\left(= \frac{8\pi}{3}\right)$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$\Rightarrow \bar{y} = \left(\text{their } \frac{8\pi}{3}\right)\times\frac{5}{8\pi}\left(= \frac{5}{3}\right)$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>Distance from plane face $= 2 - \frac{5}{3} = \frac{1}{3}$ *</td><td>A1*</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th></th><th>Mass ratio</th><th>c of m from base</th></tr>
<tr><td>S</td><td>$3\times\frac{8\pi}{5}$</td><td>$\frac{13}{3}$</td></tr>
<tr><td>cylinder</td><td>16π</td><td>2</td></tr>
<tr><td>M</td><td>$(16+4.8)\pi$</td><td>d</td></tr>
</table>
</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Moments about diameter of the base:</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$\frac{24}{5}\times\frac{13}{3} + 32 = \frac{104}{5}d\left(= \frac{264}{5}\right)$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$d = \frac{264}{104}\left(= \frac{33}{13}\right)$ (cm)</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>Complete strategy for θ</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>About to topple: $\tan\theta = \frac{2}{d}\left(= \frac{26}{33}\right)$</td><td>A1ft</td><td>3.4</td></tr>
<tr><td></td><td>$\theta = 38.2$ (or 38)</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
<tr><td></td><td colspan="3">(11 marks)</td></tr>
</table>

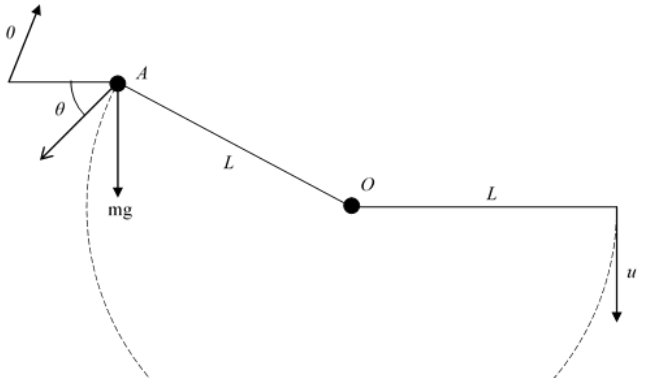
Question	Marks	Marking Guidance
5(a)	M1	Use the model to find the moment about the base. Usual rules for integration. Correct limits
	A1	Any equivalent form
	M1	Use the model and volume to find $\bar{y}$: their moments and $\frac{8\pi}{5}$ used correctly

	A1*	Obtain given answer from correct working
	(4)	
(b)	B1	Correct mass ratios seen or implied
	M1	Moments equation. Dimensionally correct. Allow for their mass ratios
	A1	Correct unsimplified equation
	A1	Any equivalent form (2.54 cm)
	M1	Complete strategy to find the position of the centre of mass of the composite body and use trig. to find θ
	A1ft	Trig ratio for a relevant angle. Follow their d
	A1	Correct angle (38 or better) (38.2338....)
	(7)	

Question	Scheme	Marks	AOs
6(a)	Use of Hooke's law seen	B1	2.1
	Let x be the displacement from equilibrium, then $T_B - T_A = -0.2\ddot{x}$.	M1	3.1a
	$\frac{15(2+x)}{1} - \frac{15(1-x)}{0.5} = -0.2\ddot{x}$	A1 A1	1.1b 1.1b
	$\Rightarrow \ddot{x} = -225x = -15^2 x$, which is of the form $\ddot{x} = -\omega^2 x$, so SHM *	A1*	3.2a
		(5)	
(b)	$a = 0.5, \quad \omega = 15$ or their ω	B1ft	1.1b
	Max speed $= a\omega = 7.5$ (m s^{-1})	B1ft	1.2
	Max acceleration $= a\omega^2 = 112.5$ (m s^{-2})	B1ft	1.2
		(3)	
(c)	$x = 0.5\cos 15t \Rightarrow \dot{x} = -7.5\sin 15t$	B1ft	2.2a
	$\lvert\dot{x}\rvert = 7.5\sin 15t = 5 \Rightarrow \sin 15t = \frac{2}{3} \Rightarrow t = ...$	M1	1.1b
	$t = 0.0486...$ (s)	A1	1.1b
	Complete strategy to find the required time	M1	3.1a
	Speed > 5 for $\frac{2\pi}{15} - 4t = 0.22$ (s)	A1	1.1b
		(5)	

Question	Marks	Marking Guidance
6(a)	B1	Use of Hooke's law seen at least once (with natural length ≤ 1.5) .
	M1	Form equation of motion involving a difference of 2 tensions. Must be dimensionally correct. Need all terms. Condone incorrect lengths. Condone sign errors.
	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	A1*	All correct and justification of SHM – comment required
	(5)	
(b)	B1ft	Correct values seen or implied. a must be correct but follow their ω from (a)
	B1ft	Follow their a, ω

	B1ft	Follow their a, ω
	(3)	
(c)	B1ft	Use correct equation for speed $\left(-a\omega\sin\omega t\right)$ Follow their a and ω Allow sin or cos form Can also use correct expression for x combined with use of $v^2=\omega^2\left(a^2-x^2\right)$
	M1	Find a relevant time e.g. time taken for 0 m s^{-1} to 5 m s^{-1}
	A1	Seen or implied
	M1	Complete strategy for the required time – e.g. find value for t when $v=5$ and use symmetry and periodic time
	A1	2 s.f. or better (0.22428…)
	(5)	**(13 marks)**

Question	Scheme	Marks	AOs
7(a)	$v=0 \Rightarrow \frac{v^2}{L}=0 \Rightarrow$ no acceleration towards O $\Rightarrow$ acceleration is perpendicular to OA	B1	2.4
		(1)	
7(b)			
	Conservation of energy	M1	2.1
	$0=\frac{1}{2}mu^2-mgL\cos\theta \quad \left(0=\frac{2gL}{3}-2gL\cos\theta\right)$	A1	1.1b
	$\Rightarrow \quad \cos\theta=\frac{1}{3}$	A1	1.1b
	Complete strategy to find the angle and |acceleration|	M1	3.1a
	Magnitude: $g\sin\theta=\frac{2\sqrt{2}}{3}g$	A1	1.1b
	$\theta=71°$ or better	A1	1.1b
		(6)	
(c)	Circular motion	M1	3.1a
	$T-mg=\frac{mv^2}{L}$	A1	1.1b
	Energy equation	M1	2.1
	$v^2=\frac{2gL}{3}+2gL\left(=\frac{8gL}{3}\right)$	A1	1.1b
	$T=mg+\frac{8mg}{3}=\frac{11mg}{3}$	A1	2.2a
		(5)	
			(12 marks)

Question	Marks	Marking Guidance
7(a)	B1	Clear explanation using $v = 0$
	(1)	
7(b)		Check their diagram to see where they have put θ .
	M1	All terms required. Must be dimensionally correct. Condone sign errors. $v = 0$ seen or implied
	A1	Correct unsimplified equation for their θ
	A1	Or equivalent to give trig ratio for relevant angle (taking account of their θ)
	M1	Complete strategy to find our θ or magnitude of acceleration
	A1	Correct magnitude from correct work only. Accept 9.2, 9.24
	A1	Correct value of θ (1.2 radians or better) from correct work only
	(6)	
(c)	M1	Equation for circular motion. Need all terms and dimensionally correct. Condone sign errors.
	A1	Correct unsimplified equation
	M1	Use of conservation of energy. Require all 3 terms and dimensionally correct.
	A1	Correct unsimplified equation
	A1	Correct only
	(5)	

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